

**MATHEMATICS
STANDARD LEVEL**

PAPER 1

Wednesday 5 May 2010 (afternoon)

Candidate session number

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

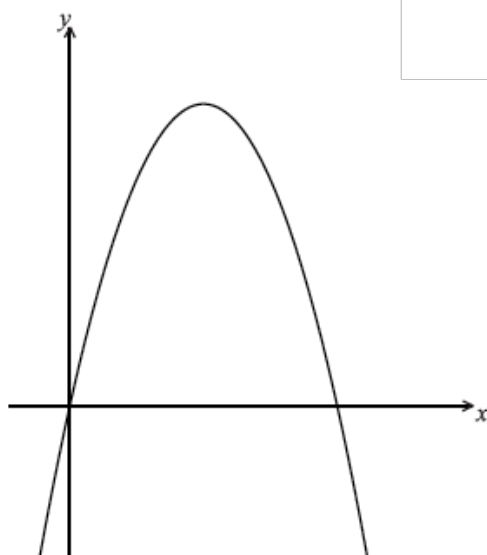
- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

SECTION A

Answer all the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.



(a) Find the x -intercepts of the graph.

[4 marks]

(b) (i) Write down the equation of the axis of symmetry.

(ii) Find the y -coordinate of the vertex.

[3 marks]

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2. [Maximum mark: 6]

Let $W = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ and $P = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find WP .

[3 marks]

(b) Given that $2WP + S = \begin{pmatrix} 26 \\ 12 \\ 10 \end{pmatrix}$, find S .

[3 marks]

3. [Maximum mark: 6]

(a) Expand $(2+x)^4$ and simplify your result. [3 marks]

(b) Hence, find the term in x^2 in $(2+x)^4 \left(1 + \frac{1}{x^2}\right)$. [3 marks]

4. [Maximum mark: 7]

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x -axis.

(a) Write down the value of $\tan \theta$. [1 mark]

(b) Find the value of

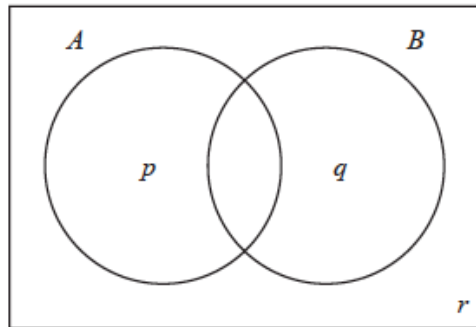
(i) $\sin 2\theta$;

(ii) $\cos 2\theta$. [6 marks]

5. [Maximum mark: 6]

Consider the events A and B , where $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B , and the probabilities p , q and r .



(a) Write down the value of

(i) p ;

(ii) q ;

(iii) r .

[3 marks]

(b) Find the value of $P(A | B')$.

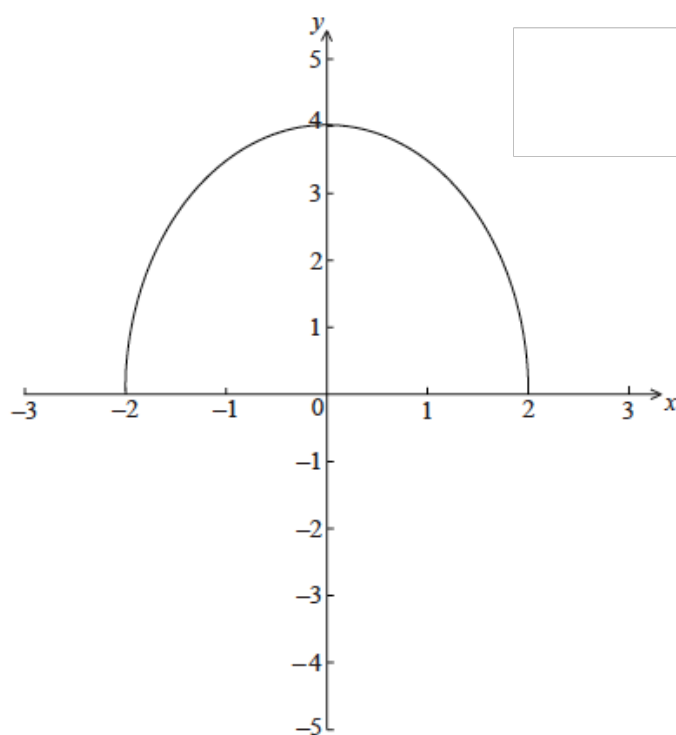
[2 marks]

(c) Hence, or otherwise, show that the events A and B are **not** independent.

[1 mark]

6. [Maximum mark: 6]

The graph of $f(x) = \sqrt{16 - 4x^2}$, for $-2 \leq x \leq 2$, is shown below.



The region enclosed by the curve of f and the x -axis is rotated 360° about the x -axis.
Find the volume of the solid formed.

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7. [Maximum mark: 7]

Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$. [2 marks]

(b) Write down the range of f^{-1} . [1 mark]

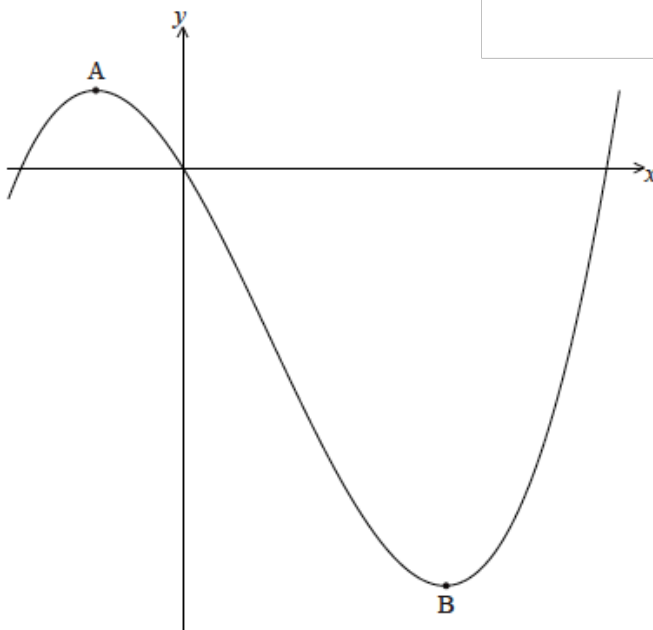
Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer. [4 marks]

Answer all the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 14]

Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown



There is a maximum point at A and a minimum point at $B(3, -9)$.

(a) Find the coordinates of A. [8 marks]

(b) Write down the coordinates of

(i) the image of B after reflection in the y -axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x -axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

[6 marks]

9. [Maximum mark: 13]

Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

(a) Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. [5 marks]

(b) Find $f''(x)$. [3 marks]

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2} - 0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.1$
$f'(x)$	-1.01	p	-1.01
$f''(x)$	0.203	q	-0.203

(c) Find the value of p and of q . [3 marks]

(d) Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$. [2 marks]

10. [Maximum mark: 18]

The line L_1 is represented by the vector equation $\mathbf{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point $B(-8, -5, 25)$.

- (a) Write down a vector equation for L_2 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2 marks]

A third line L_3 is perpendicular to L_1 and is represented by $\mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

- (b) Show that $k = -2$. [5 marks]

The lines L_1 and L_3 intersect at the point A.

- (c) Find the coordinates of A. [6 marks]

The lines L_2 and L_3 intersect at point C where $\vec{BC} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

- (d) (i) Find \vec{AB} .

- (ii) Hence, find $|\vec{AC}|$. [5 marks]